

FINITE DIFFERENCE ANALYSIS OF ROTORDYNAMIC SEAL
COEFFICIENTS FOR AN ECCENTRIC SHAFT POSITION

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The dynamic coefficients of seals are calculated for shaft movements around an eccentric position. The turbulent flow is described by the Navier-Stokes equations in connection with a turbulence model. The equations are solved by a finite-difference procedure.

INTRODUCTION

To model the dynamic behaviour of turbopumps properly it is very important to consider the fluid forces which are developed in the seals. This has been clearly demonstrated by some authors, like for example Diewald (1987). The fluid forces are normally described by the following equation.

$$-\begin{bmatrix} F_z \\ F_y \end{bmatrix} = \begin{bmatrix} K & k \\ -k & K \end{bmatrix} \begin{bmatrix} z \\ y \end{bmatrix} + \begin{bmatrix} D & d \\ -d & D \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{y} \end{bmatrix} \quad (1)$$

But this equation is only valid for a shaft moving around the center of the seal, which very seldom occurs in reality. In most machines the shaft will orbit around an eccentric position, so that the fluid forces must be described by

$$-\begin{bmatrix} F_z \\ F_y \end{bmatrix} = \begin{bmatrix} K_{zz} & k_{zy} \\ -k_{yz} & K_{yy} \end{bmatrix} \begin{bmatrix} Z \\ Y \end{bmatrix} + \begin{bmatrix} D_{zz} & d_{zy} \\ -d_{yz} & D_{yy} \end{bmatrix} \begin{bmatrix} \dot{Z} \\ \dot{Y} \end{bmatrix} + \begin{bmatrix} M_{zz} & 0 \\ 0 & M_{yy} \end{bmatrix} \begin{bmatrix} \ddot{Z} \\ \ddot{Y} \end{bmatrix} \quad (2)$$

The dynamic coefficients in such a case have been investigated by Jenssen (1970), Allaire et al. (1976) and recently in an excellent paper by Nelson and Nguyen (1987). To model the turbulent flow, all these have used so called "Bulk-Flow Theories" in which the shear stress at the wall is described as a function of the average fluid velocity relative to the wall. Because some authors affirm that in the case of great eccentricities recirculation in circumferential direction occurs (which can't be described by a bulk-flow model) and strongly effects the dynamic coefficients, we extended the theory of Dietzen and Nordmann (1987) and Nordmann (1987) to investigate the flow and the coefficients in the case of an eccentric shaft. In that theory the Navier Stokes equations in connection with the k-ε turbulence model were used to calculate the dynamic coefficients of incompressible and compressible seals for a shaft motion around the centric position.

GOVERNING EQUATIONS

In a turbulent flow, turbulent stresses occur which are often modeled like laminar stresses by introducing a turbulent viscosity. The turbulent and the laminar viscosity are then added to an effective viscosity.

$$\mu_e = \mu_l + \mu_t \quad (3)$$

The turbulent viscosity must be described by a turbulence model. We use the k- ϵ model, but also much simpler mixing-length models will be appropriate for a straight seal. The turbulent viscosity is given by

$$\mu_t = C_\mu \rho \frac{k^2}{\epsilon} \quad (4)$$

So we have the Navier-Stokes equations, the continuity equation and the equations of the k- ϵ model to describe the turbulent flow in a seal. These equations have the following form.

$$\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial}{\partial x}(\rho u\phi) - \frac{\partial}{\partial x}(\Gamma \frac{\partial\phi}{\partial x}) + \frac{1}{r} \frac{\partial}{\partial r}(r\rho v\phi) - \frac{1}{r} \frac{\partial}{\partial r}(r\Gamma \frac{\partial\phi}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\rho w\phi) - \frac{1}{r} \frac{\partial}{\partial \theta}(\Gamma \frac{1}{r} \frac{\partial\phi}{\partial \theta}) = S_\phi \quad (5)$$

ϕ	Γ	S_ϕ
u	μ_e	$-\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(\mu_e \frac{\partial u}{\partial x}) + \frac{1}{r} \frac{\partial}{\partial r}(r\mu_e \frac{\partial v}{\partial x}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\mu_e \frac{\partial w}{\partial x})$
v	μ_e	$-\frac{\partial p}{\partial r} + \frac{\partial}{\partial x}(\mu_e \frac{\partial u}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial r}(r\mu_e \frac{\partial v}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial \theta}(r\mu_e \frac{\partial}{\partial r}(\frac{w}{r}))$ $-\frac{2}{r^2} \mu_e \frac{\partial w}{\partial \theta} - \frac{2}{r^2} \mu_e v + \frac{\rho}{r} w^2$
w	μ_e	$-\frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{\partial}{\partial x}(\frac{1}{r} \mu_e \frac{\partial u}{\partial \theta}) + \frac{1}{r} \frac{\partial}{\partial r}(\mu_e \frac{\partial v}{\partial \theta}) + \frac{1}{r} \frac{\partial}{\partial \theta}(\frac{1}{r} \mu_e \frac{\partial w}{\partial \theta})$ $+ \frac{1}{r^2} \mu_e \frac{\partial v}{\partial \theta} - \frac{w}{r^2} \frac{\partial}{\partial r}(r\mu_e) + \frac{1}{r} \frac{\partial}{\partial \theta}(\frac{2}{r} \mu_e v) - \frac{\rho}{r} vw$
1	0	0
k	μ_e / σ_k	$G - \rho \epsilon$
ϵ	μ_e / σ_ϵ	$C_1 \frac{\epsilon}{k} G - C_2 \rho \frac{\epsilon^2}{k}$

Table 1: The governing equations of the turbulent seal flow.

(The constants of the k- ϵ model are given in appendix A)

PERTURBATION ANALYSIS

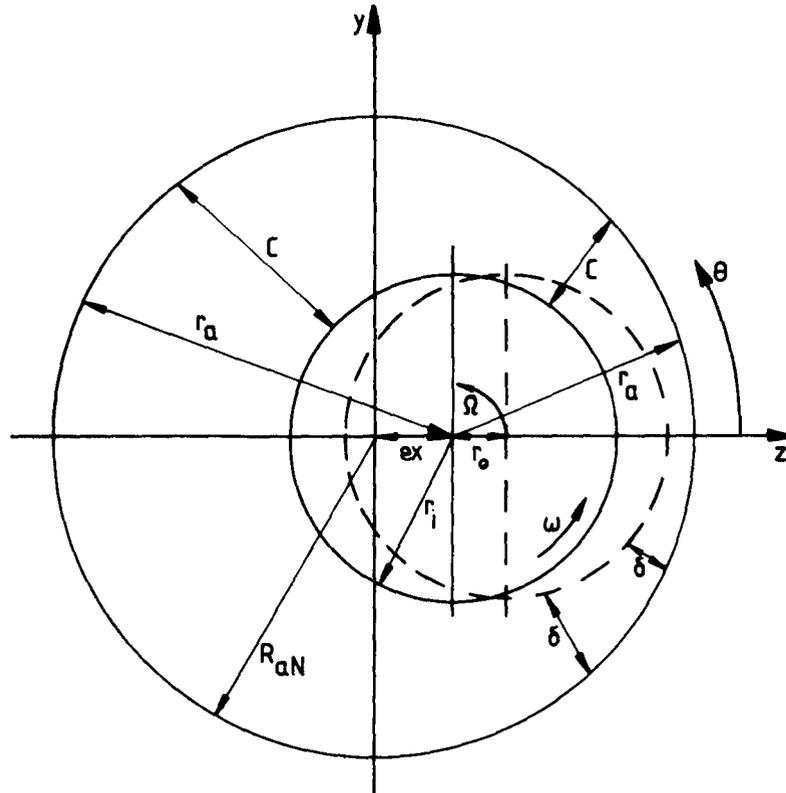


Fig. 1 Geometry of the eccentric shaft

To describe the flow if the shaft is moving around an eccentric position we follow the procedure of Dietzen and Nordmann (1987) and use a similar transformation. (see Fig. 1)

$$\eta = r_a(\theta) - \frac{r_a(\theta) - r}{\delta(\theta, t)} C_o(\theta) \quad (6)$$

But now r_a and C_o are functions of θ . This is not so if the shaft moves around the center. δ is the seal clearance, varying with angle θ and time t .

The radius r_a can be described by the following equation:

$$r_a(\theta) = \sqrt{R_{aN}^2 - ex^2 \sin^2(\theta)} - ex \cos(\theta) \quad (7)$$

As result of this transformation the shaft orbiting around an eccentric position is transformed to the stationary eccentric position.

If we introduce this transformation into our equations, we must obey the following relations.

$$\left(\frac{\partial}{\partial \theta}\right)_r = \left(\frac{\partial}{\partial \theta}\right)_\eta + \left(\frac{\partial}{\partial \eta}\right)_\theta \left(\frac{\partial \eta}{\partial \theta}\right)_r \quad \left(\frac{\partial}{\partial t}\right)_r = \left(\frac{\partial}{\partial t}\right)_\eta + \left(\frac{\partial}{\partial \eta}\right)_t \left(\frac{\partial \eta}{\partial t}\right)_r \quad (8)$$

To calculate the rotordynamic coefficients we assume that the shaft moves around the eccentric position on small orbits, so that we are allowed to introduce a perturbation analysis.

$$\begin{aligned} \delta &= C_0 - eh_1 & u &= u_0 + eu_1 & v &= v_0 + ev_1 & (9) \\ & & w &= w_0 + ew_1 & p &= p_0 + ep_1 \end{aligned}$$

If we introduce these expressions and the coordinate transformation in our governing equation, neglecting terms with power of e greater than 1 and separating the parts with and without e we will get a set of zeroth order and first order equations. The zeroth order equations describe the stationary flow for the eccentric shaft, the first order equations the perturbation of this flow, if the shaft moves around the stationary position.

We assume that the shaft moves with the precession frequency Ω on a circular orbit with radius r_0 around the eccentric position. So the change of the clearance is given by

$$eh_1 = r_0(\cos\Omega t \cos\theta + \sin\Omega t \sin\theta) \quad (10)$$

Because this change is periodic in time we introduce also periodic functions for the flow variables.

$$\begin{aligned} u_1 &= u_{1c} \cos\Omega t + u_{1s} \sin\Omega t & v_1 &= v_{1c} \cos\Omega t + v_{1s} \sin\Omega t & (11) \\ w_1 &= w_{1c} \cos\Omega t + w_{1s} \sin\Omega t & p_1 &= p_{1c} \cos\Omega t + p_{1s} \sin\Omega t \end{aligned}$$

By separating now in the first order equations the terms with $\cos\Omega t$ and $\sin\Omega t$ we obtain two real equations for every first order equation. These equations are then arranged in a new form by introducing complex variables.

$$\begin{aligned} \hat{u}_1 &= u_{1c} + iu_{1s} & \hat{v}_1 &= v_{1c} + iv_{1s} & (12) \\ \hat{w}_1 &= w_{1c} + iw_{1s} & \hat{p}_1 &= p_{1c} + ip_{1s} \end{aligned}$$

Finally supplementary to our real zeroth order equations we have a set of complex first order equations. These equations have the following form.

$$\frac{\partial}{\partial x}(\rho u_0 \phi) - \frac{\partial}{\partial x}(r \frac{\partial \phi}{\partial x}) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \rho v_0 \phi) - \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta r \frac{\partial \phi}{\partial \eta}) + \frac{1}{\eta} \frac{\partial}{\partial \theta}(\rho w_0 \phi) - \frac{1}{\eta} \frac{\partial}{\partial \theta}(r \frac{1}{\eta} \frac{\partial \phi}{\partial \theta}) = S_\phi \quad (13)$$

ϕ	Γ_ϕ	S_ϕ
u_0	μ_e	S_{u_0}
v_0	μ_e	S_{v_0}
w_0	μ_e	S_{w_0}
1	0	0
k_0	μ_e/σ_k	S_{k_0}
ϵ_0	μ_e/σ_ϵ	S_{ϵ_0}
\hat{u}_1	μ_e	$-\frac{\hat{\partial} p_1}{\partial x} + \frac{\partial}{\partial x}(\mu_e \frac{\hat{\partial} u_1}{\partial x}) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \mu_e \frac{\hat{\partial} v_1}{\partial x}) + \frac{1}{\eta} \frac{\partial}{\partial \theta}(\mu_e \frac{\hat{\partial} w_1}{\partial x})$ $- C_{u_1} + i\rho\Omega u_1 + D_{u_0}$
\hat{v}_1	μ_e	$-\frac{\hat{\partial} p_1}{\partial \eta} + \frac{\partial}{\partial x}(\mu_e \frac{\hat{\partial} u_1}{\partial \eta}) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \mu_e \frac{\hat{\partial} v_1}{\partial \eta})$ $+ \frac{1}{\eta} \frac{\partial}{\partial \theta}(\eta \mu_e \frac{\hat{\partial} w_1}{\partial \eta}) - \frac{2}{\eta^2} \mu_e \frac{\hat{\partial} w_1}{\partial \theta} - \frac{2}{\eta^2} \mu_e \hat{v}_1 + \frac{\rho}{\eta} w_0 \hat{w}_1$ $- C_{v_1} + i\rho\Omega v_1 + D_{v_0}$
\hat{w}_1	μ_e	$-\frac{1}{\eta} \frac{\hat{\partial} p_1}{\partial \theta} + \frac{\partial}{\partial x}(\frac{1}{\eta} \mu_e \frac{\hat{\partial} u_1}{\partial \theta}) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\mu_e \frac{\hat{\partial} v_1}{\partial \theta}) + \frac{1}{\eta} \frac{\partial}{\partial \theta}(\frac{1}{\eta} \mu_e \frac{\hat{\partial} w_1}{\partial \theta})$ $+ \frac{1}{\eta^2} \mu_e \frac{\hat{\partial} v_1}{\partial \theta} - \frac{\hat{w}_1}{\eta^2} \frac{\partial}{\partial \eta}(\eta \mu_e) + \frac{1}{\eta} \frac{\partial}{\partial \theta}(\frac{2}{\eta} \mu_e \hat{v}_1) - \frac{\rho}{\eta} v_0 \hat{w}_1$ $- C_{w_1} + i\rho\Omega w_1 + D_{w_0}$

Table 2: Source terms of zeroth and first order equations.

Only the first order continuity equation to calculate \hat{p}_1 has a slightly different form.

$$\frac{\partial}{\partial x}(\rho \hat{u}_1) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \rho \hat{v}_1) + \frac{1}{\eta} \frac{\partial}{\partial \theta}(\rho \hat{w}_1) = D_{p_0} \quad (14)$$

You get S_{u_0} , S_{v_0} , S_{w_0} , S_{k_0} , S_{ϵ_0} , if you replace in table 1 in the corresponding terms r by η and u, v, w, p, k, ϵ by $u_0, v_0, w_0, p_0, k_0, \epsilon_0$. The terms C_{u_1} , C_{v_1} , C_{w_1} result from the perturbation of the convective terms in the Navier Stokes equations. D_{u_0} , D_{v_0} , D_{w_0} , D_{p_0} are constants resulting from the coordinate transformation, which are not functions of \hat{u}_1 , \hat{v}_1 , \hat{w}_1 , \hat{p}_1 . The terms with Ω represent the time dependent parts.

Because we assume that the viscosity μ_e remains constant for the small motions, we do not need a \hat{k}_1 and $\hat{\epsilon}_1$ equation.

(D_{u_0} , D_{v_0} , D_{w_0} , D_{p_0} , C_{u_1} , C_{v_1} , C_{w_1} are given in appendix B)

BOUNDARY CONDITIONS

The zeroth order boundary conditions are

$$\begin{aligned} \text{stator} & : u_{0s} = 0 & v_{0s} = 0 & w_{0s} = 0 \\ \text{rotor} & : u_{0r} = 0 & v_{0r} = 0 & w_{0r} = R_{iN}\omega \\ \text{entrance} & : p_{0En} = p_{Res} - \frac{1}{2}\rho u_{0En}^2(1 + \xi) \\ \text{exit} & : p_{0Ex} = 0 \end{aligned}$$

u_{En} is the average axial entrance velocity for every plane with θ constant. p_{Res} is the reservoir pressure and ω the rotational frequency of the shaft.

The first order boundary conditions are

$$\begin{aligned} \text{stator} & : \hat{u}_{1s} = (0,0) & \hat{v}_{1s} = (0,0) & \hat{w}_{1s} = (0,0) \\ \text{rotor} & : \hat{u}_{1r} = (0,0) \\ & \hat{v}_{1r} = [C_{0N}(\Omega - \omega) \sin\theta, -C_{0N}(\Omega - \omega) \cos\theta] \\ & \hat{w}_{1r} = [C_{0N}\Omega \cos\theta, C_{0N}\Omega \sin\theta] \\ \text{entrance} & : \hat{p}_{1En} = -\rho u_{0En}(1 + \xi) \hat{u}_{1En} \\ \text{exit} & : \hat{p}_{1Ex} = (0,0) \end{aligned}$$

THE FINITE DIFFERENCE METHOD

For solving these equations a finite-difference procedure is used which is based on the method published by Gosman and Pun (1974). The seal is discretized by a grid (Fig.2) and the variables are calculated at the nodes. The velocities u_0, v_0, w_0 ($\hat{u}_1, \hat{v}_1, \hat{w}_1$) are determined at points which lie between the nodes where the variables p_0, k_0, ϵ_0 (\hat{p}_1) are calculated (Fig.3)

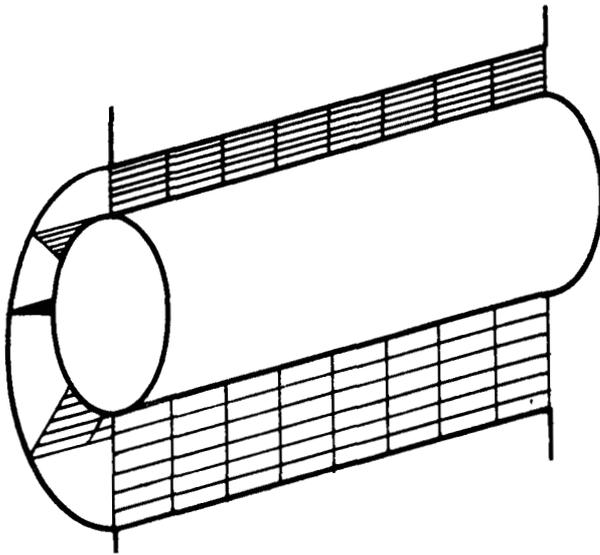


Fig. 2 3-dimensional grid in the seal.

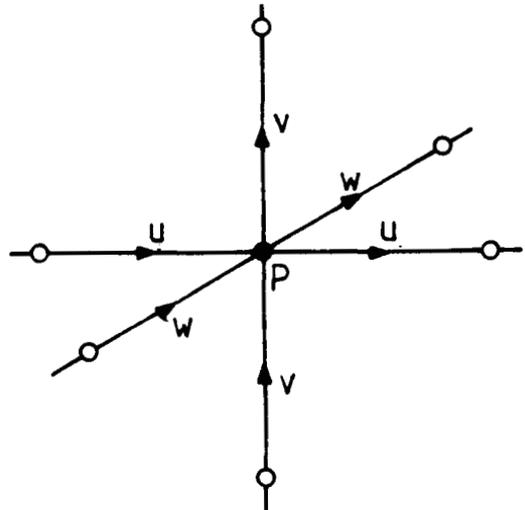


Fig. 3 Velocities and pressure in a staggered grid.

To calculate the flow we proceed as follows:

1. We start our procedure with guessed values for all variables.
2. First the velocities u_0, v_0, w_0 ($\hat{u}_1, \hat{v}_1, \hat{w}_1$) are calculated.
3. Then the velocities and the pressure are corrected to satisfy the continuity equation. To do that we use a modified version of the 'PISO' procedure of Benodekar et al. (1985).
4. After this k and ϵ are calculated (only for the zeroth order solution).

We repeat step 2 to 4 until we reach a convergent solution. First we solve the zeroth order equations and then the first order equations.

Of course we need a 3-dimensional finite-difference method to calculate the flow in the case of an eccentric shaft, while a two-dimensional method is sufficient for movements around the centric position.

DETERMINATION OF THE DYNAMIC COEFFICIENTS

By integration the pressure \hat{p}_1 around the shaft we get forces in z and y directions. Then we introduce $z, y, \dot{z}, \dot{y}, \ddot{z}, \ddot{y}$ from our circular orbit into equation (2). This gives us the following equations:

$$\int_0^L \int_0^{2\pi} p_{1c} \cos\theta R_{iN} d\theta dx = C_{oN} (K_{zz} + d_{zy}\Omega - M_{zz}\Omega^2)$$

$$\int_0^L \int_0^{2\pi} p_{1s} \cos\theta R_{iN} d\theta dx = C_{oN} (k_{zy} - D_{zz}\Omega)$$

$$\int_0^L \int_0^{2\pi} p_{1c} \sin\theta R_{iN} d\theta dx = C_{oN} (-k_{yz} + D_{yy}\Omega)$$

$$\int_0^L \int_0^{2\pi} p_{1s} \sin\theta R_{iN} d\theta dx = C_{oN} (K_{yy} + d_{yz}\Omega - M_{yy}\Omega^2)$$

If we calculate the forces for several precession frequencies Ω of the shaft, we can obtain the coefficients by a 'Least-Square-Fit'.

RESULTS

We compare our theory with the model of Nelson and Nguyen (1987) and some experimental results of Falco et al. (1986) which also have been published in the paper of Nelson and Nguyen (1987).

In Fig. 4-10 dynamic coefficients are calculated as a function of the eccentricity. We compare our results with Nelson and Nguyen's theory and the stiffness coefficients also with experimental and theoretical results of Falco et al.

The seal data are

length	: L = 40.0 mm	pressure drop	: 1.0 Mpa
shaft radius	: $R_{iN} = 80.0$ mm	shaft speed	: 4000 RPM
nominal clearance	: $C_{oN} = 0.36$ mm		
density	: $\rho = 1000$ kg/m ³		
preswirl ratio	: $\bar{w}(0, \theta)/R_{iN}\omega = 0.3$		
viscosity	: $\mu_1 = 1.0 \times 10^{-3}$ Ns/m ²		
entrance lost-coefficient	: $\xi = 0.5$		

CONCLUSIONS

We have shown that it is possible to calculate the dynamic coefficients of eccentric seals by a finite difference method based on the Navier Stokes equations. This method can also be extended to calculate the coefficients of eccentric gas seals by following the procedure of Nordmann (1987) and to calculate the dynamic coefficients of bearings by neglecting the turbulence model.

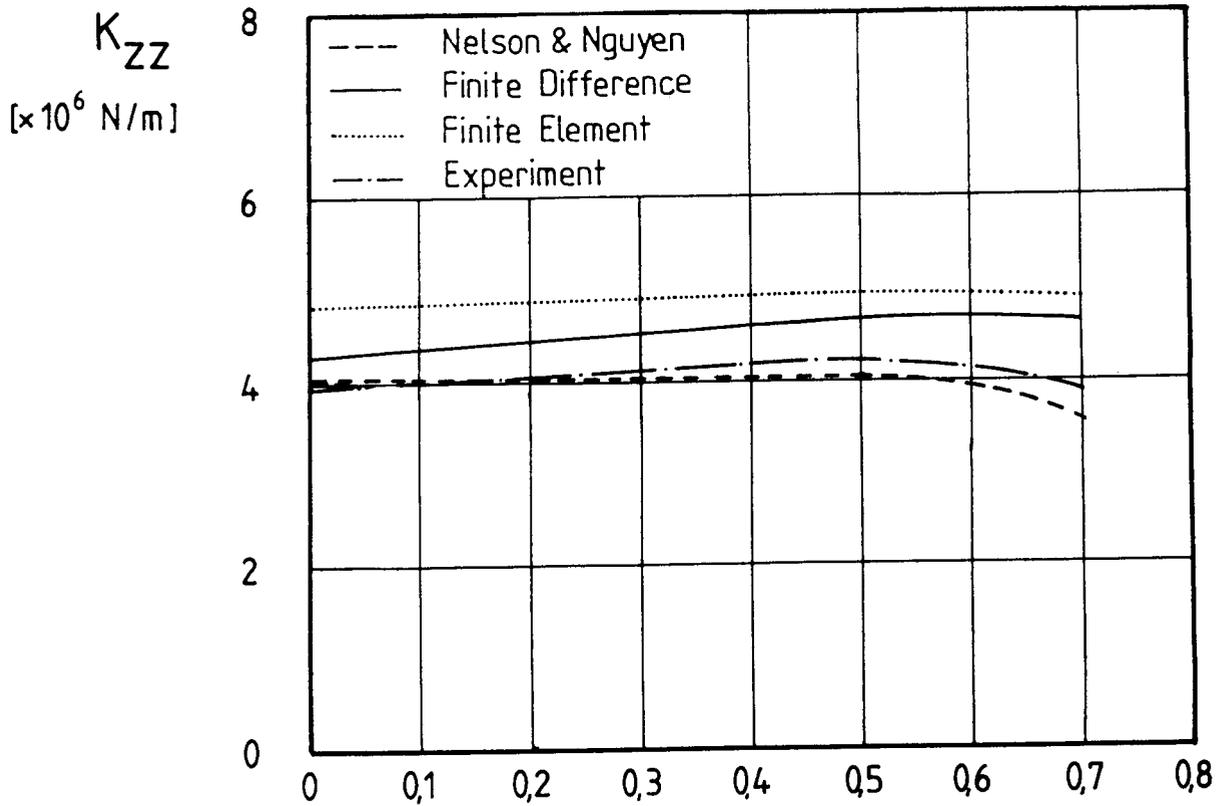


Fig. 4 Direct stiffness K_{zz} versus eccentricity.

ex/c_0

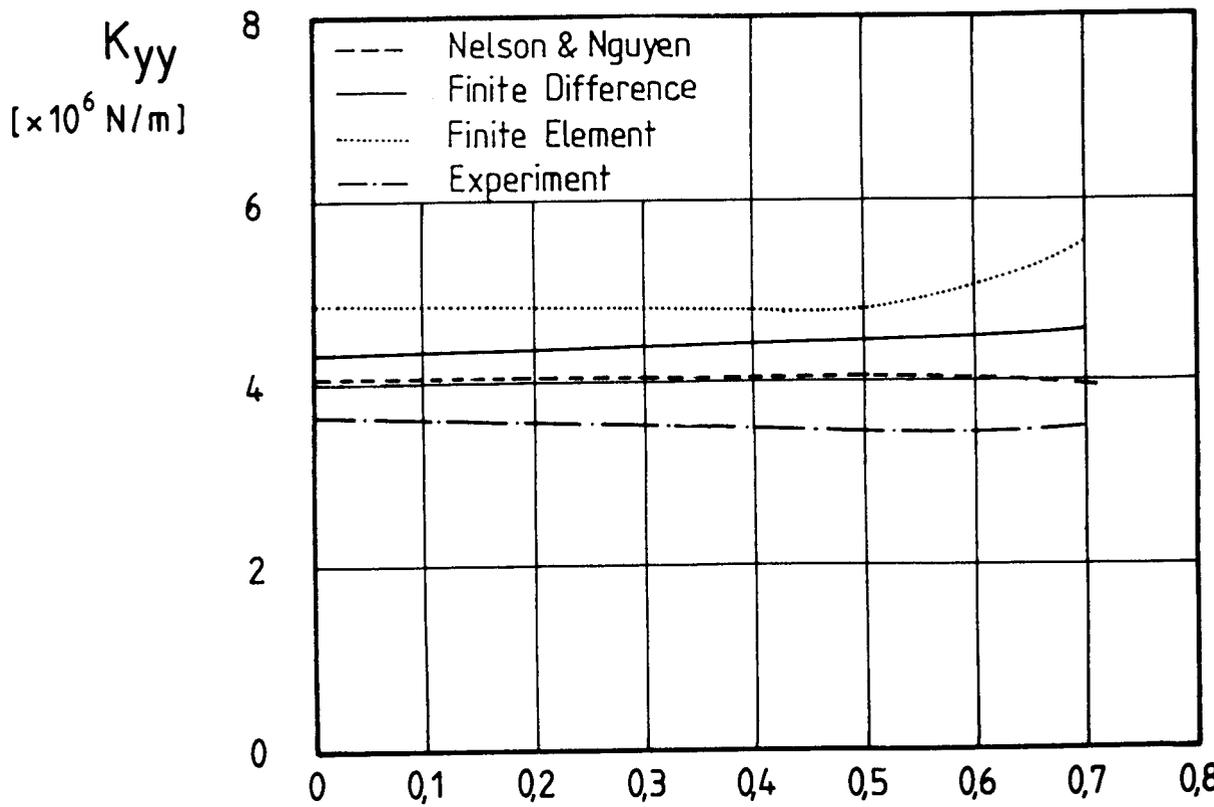


Fig. 5 Direct stiffness K_{yy} versus eccentricity.

ex/c_0

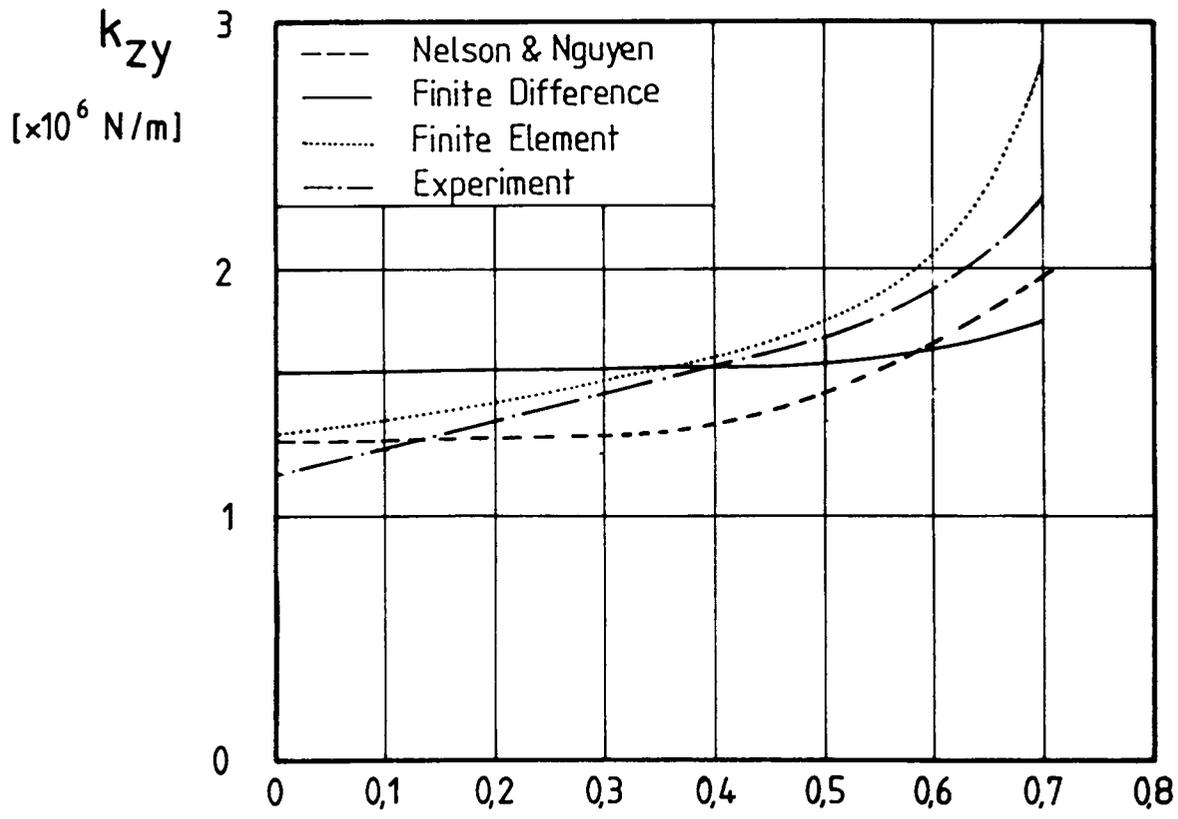


Fig. 6 Cross coupled stiffness k_{zy} versus eccentricity.

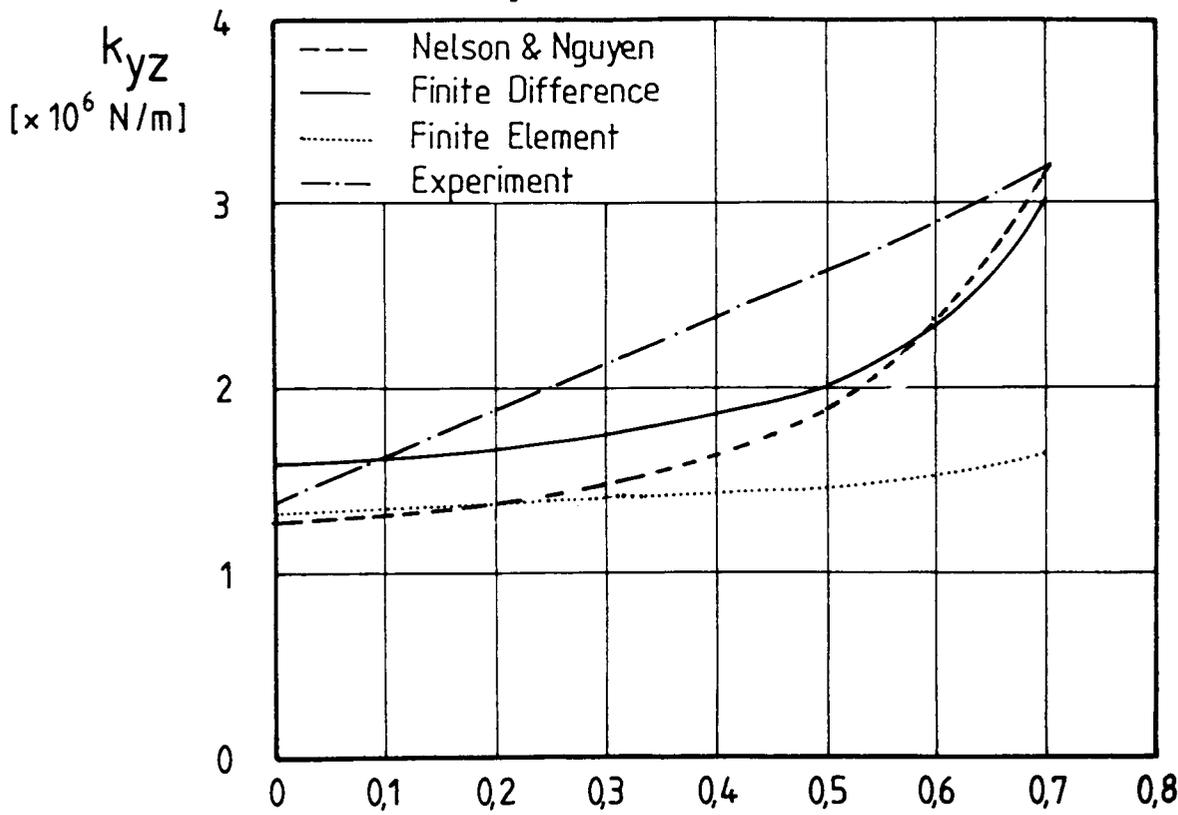


Fig. 7 Cross coupled stiffness k_{yz} versus eccentricity.

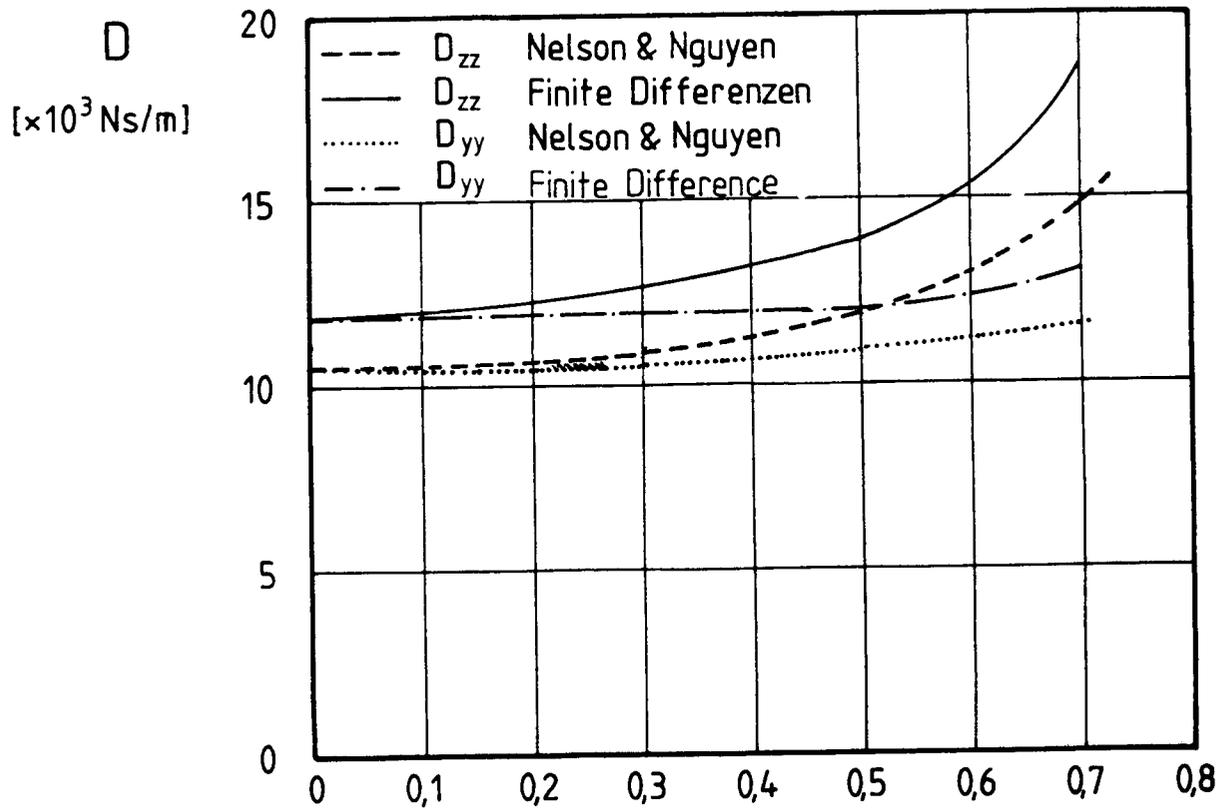


Fig. 8 Direct damping versus eccentricity.

ex/c_0

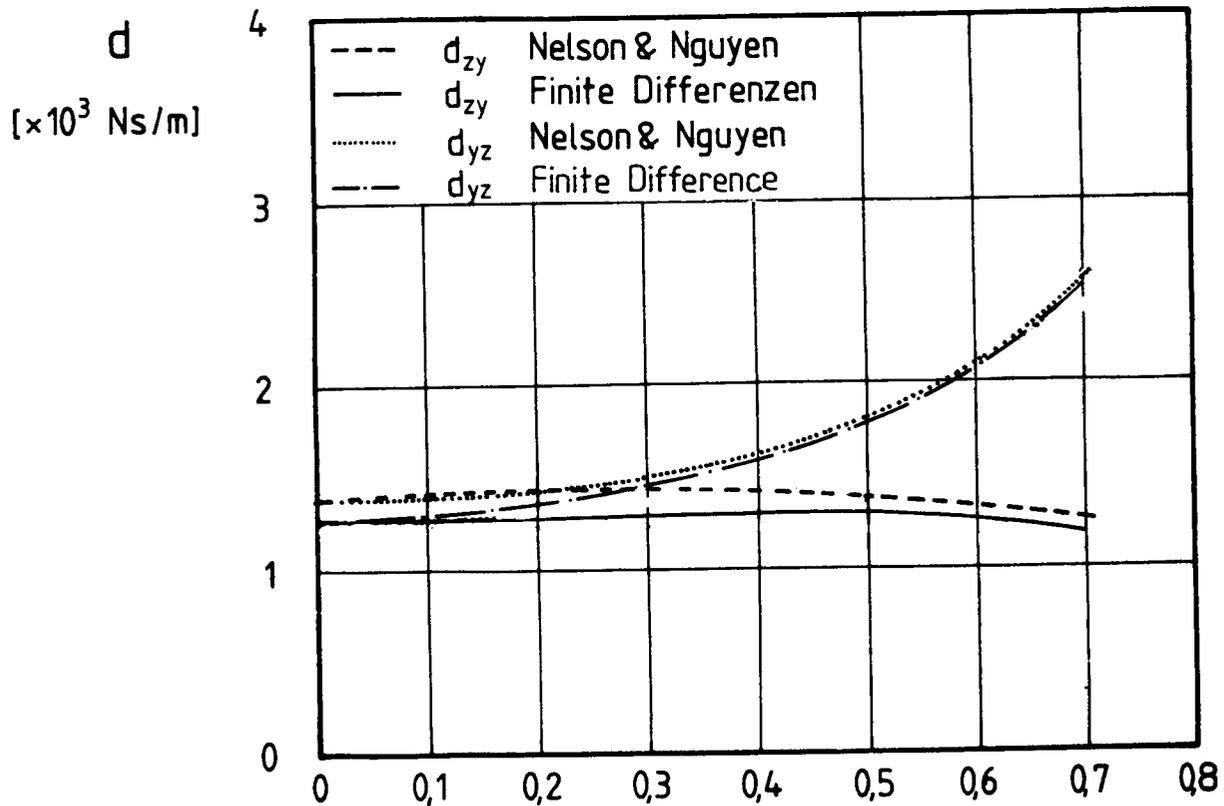


Fig. 9 Cross coupled damping versus eccentricity.

ex/c_0

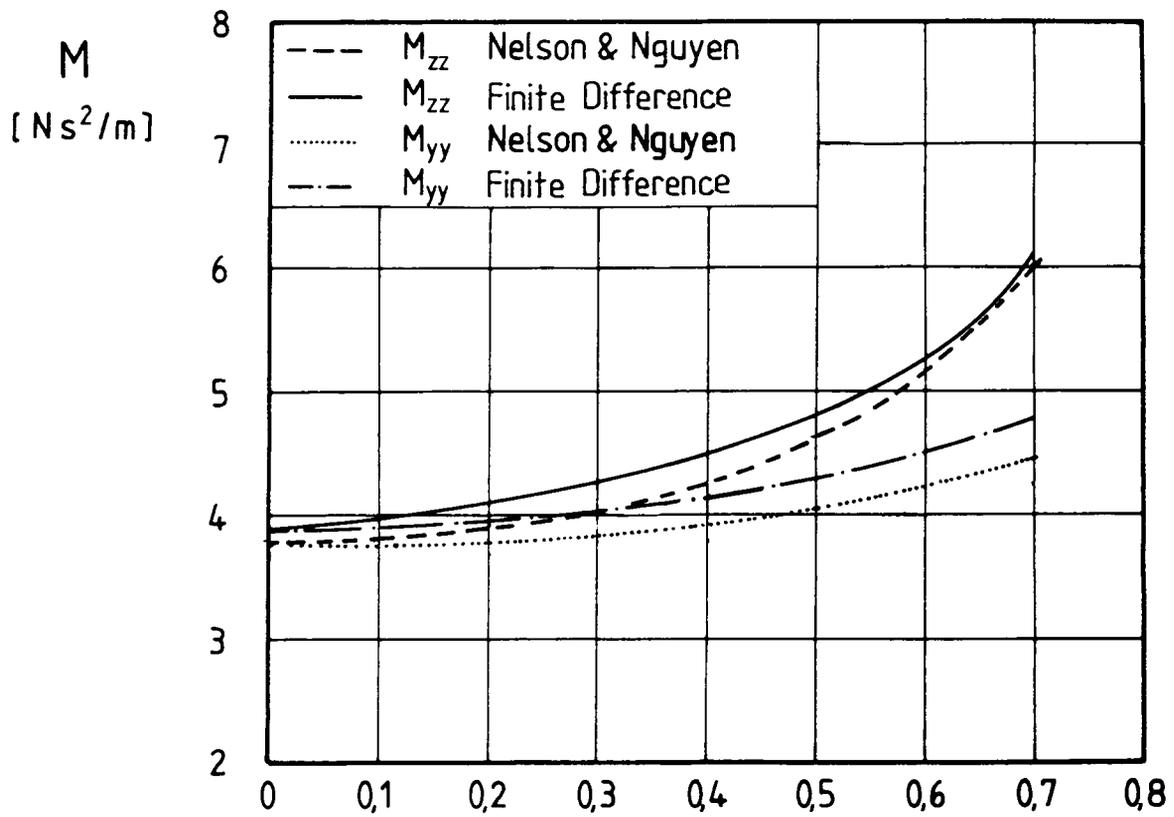


Fig. 10 Direct inertia versus eccentricity.

ex/c_0

NOMENCLATURE

F_z, F_y	Forces on the shaft in z and y direction
K, k	direct and cross-coupling stiffness
D, d	direct and cross-coupled damping
M, m	direct and cross-coupling inertia
u, v, w	axial, radial and circumferential velocity
p	pressure
k	turbulence energy
ϵ	energy dissipation
μ_e, μ_l, μ_t	effective, laminar and turbulent viscosity
ρ	density
t	time
x, r, θ	axial, radial and circumferential coordinate
x, y, z	rectangular coordinate directions
Y, Z	rotor displacements from its steady-state position
η	radial coordinate after transformation
G	production term in k- ϵ -model
$\sigma_k, \sigma_\epsilon$	constants of the k- ϵ -model
C_μ, C_1, C_2	constants of the k- ϵ -model
ϕ	general variable standing for u, v, w, p, k ϵ
S_ϕ	general source term
C_o	nominal seal clearance
C_{oN}	steady-state clearance for an eccentric shaft
δ	varying seal clearance for orbiting shaft
r_o	radius of the precession motion of the shaft
$e = \frac{r_o}{C_{oN}}$	perturbation parameter
ω	rotational frequency of the shaft = RPM $\times\pi/30$
RPM	revolutions per minute
Ω	precession frequency of the shaft
ξ	entrance lost-coefficient
L	Length of the seal
r_a	stator radius (Fig. 1)
R_{aN}	nominal stator radius (Fig. 1)
R_{iN}	shaft radius (Fig. 1)
h_1	change in the clearance
ex	eccentricity
$-$	
w	average circumferential velocity
u_{En}	average axial velocity at the entrance
P_{Res}	sump pressure

APPENDIX A: Constants of the k-ε model.

$$C_{\mu} = 0.09 \quad C_1 = 1.44 \quad C_2 = 1.92 \quad \sigma_k = 1.0 \quad \sigma_{\epsilon} = 1.3$$

and G is a given by

$$G = \mu_e \left[2 \left(\left(\frac{\partial v}{\partial r} \right)^2 + \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{1}{r} \frac{\partial w}{\partial \theta} + \frac{v}{r} \right)^2 \right) + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial r} \right) + \left(\frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r} \right)^2 + \left(\frac{\partial w}{\partial x} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right)^2 \right]$$

APPENDIX B: Terms of first order source term.

$$C_{u_1} = \frac{\partial}{\partial x}(\rho u_0 \hat{u}_1) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \rho u_0 \hat{v}_1) + \frac{1}{\eta} \frac{\partial}{\partial \theta}(\rho u_0 \hat{w}_1)$$

$$C_{v_1} = \frac{\partial}{\partial x}(\rho v_0 \hat{u}_1) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \rho v_0 \hat{v}_1) + \frac{1}{\eta} \frac{\partial}{\partial \theta}(\rho v_0 \hat{w}_1) - \frac{\rho w_0 \hat{w}_1}{\eta}$$

$$C_{w_1} = \frac{\partial}{\partial x}(\rho w_0 \hat{u}_1) + \frac{1}{\eta} \frac{\partial}{\partial \eta}(\eta \rho w_0 \hat{v}_1) + \frac{1}{\eta} \frac{\partial}{\partial \theta}(\rho w_0 \hat{w}_1) + \frac{\rho w_0 \hat{v}_1}{\eta}$$

$$D_{u_0} = (((r_a - \eta)DFC + RCS)U1 + CCC * U2 + OCS * U3) + i(((r_a - \eta)DFS + RSS)U1 + OCS * U2 + CCC * U3)$$

$$D_{v_0} = (((r_a - \eta)DFC + RCS)V1 + CCC * V2 + OCS * V3) + i(((r_a - \eta)DFS + RSS)V1 + OCS * V2 + CCC * V3)$$

$$D_{w_0} = (((r_a - \eta)DFC + RCS)W1 + CCC * W2 + OCS * W3) + i(((r_a - \eta)DFS + RSS)W1 + OCS * W2 + CCC * W3)$$

$$D_{p_0} = (((r_a - \eta)DFC + RCS)P1 - CCC * P2) + i(((r_a - \eta)DFS + RSS)P1 - OCS * P2)$$

$$DFS = \frac{C_{0N}}{C_0} \left[\cos(\theta) - \frac{ex}{C_0} * \sin(\theta) \sin(\theta) * \left(1 - \frac{ex \cos(\theta)}{\sqrt{R_{aN}^2 - ex^2 \sin^2(\theta)}} \right) \right]$$

$$DFC = - \frac{C_{0N}}{C_0} \left[\sin(\theta) + \frac{ex}{C_0} * \cos(\theta) \sin(\theta) * \left(1 - \frac{ex \cos(\theta)}{\sqrt{R_{aN}^2 - ex^2 \sin^2(\theta)}} \right) \right]$$

$$RCS = ex \frac{C_{0N}}{C_0} \left[\cos(\theta) \sin(\theta) * \left(1 - \frac{ex \cos(\theta)}{\sqrt{R_{aN}^2 - ex^2 \sin^2(\theta)}} \right) \right]$$

$$RSS = ex \frac{C_{0N}}{C_0} \left[\sin(\theta) \sin(\theta) * \left(1 - \frac{ex \cos(\theta)}{\sqrt{R_{aN}^2 - ex^2 \sin^2(\theta)}} \right) \right]$$

$$\text{OCC} = \frac{C_{oN}}{C_o} \cos(\theta) \quad \text{OCS} = \frac{C_{oN}}{C_o} \sin(\theta)$$

$$\text{OCC} = -\Omega \frac{C_{oN}}{C_o} \cos(\theta) \quad \text{OCS} = \Omega \frac{C_{oN}}{C_o} \sin(\theta)$$

$$P1 = \frac{1}{\eta} \frac{\partial}{\partial \eta} (\rho w_o)$$

$$P2 = \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \rho v_o) + \left(1 - \frac{r_a}{\eta}\right) \frac{1}{\eta} \frac{\partial}{\partial \theta} (\rho w_o) - \frac{r_a}{\eta^2} \rho v_o$$

$$U1 = \frac{1}{\eta} \left[\frac{\partial}{\partial \eta} (\rho u_o w_o) - \frac{\partial}{\partial \eta} \left(\frac{\mu_e}{\eta} \frac{\partial u_o}{\partial \theta} \right) - \frac{\partial}{\partial \eta} \left(\mu_e \frac{\partial w_o}{\partial x} \right) \right]$$

$$U2 = \frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\eta \mu_e \frac{\partial v_o}{\partial x} + \eta \mu_e \frac{\partial u_o}{\partial \eta} \right) - \frac{r_a}{\eta^2} \mu_e \frac{\partial v_o}{\partial x} - \frac{r_a}{\eta^2} \mu_e \frac{\partial u_o}{\partial \eta} + \left(1 - \frac{r_a}{\eta}\right) \frac{1}{\eta} \frac{\partial}{\partial \theta} \left(\frac{\mu_e}{\eta} \frac{\partial u_o}{\partial \theta} + \mu_e \frac{\partial w_o}{\partial x} \right) \\ - \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \rho v_o u_o) + \frac{r_a}{\eta^2} \rho v_o u_o - \left(1 - \frac{r_a}{\eta}\right) \frac{1}{\eta} \frac{\partial}{\partial \theta} (\rho u_o w_o)$$

$$U3 = (r_a - \eta) \frac{\partial}{\partial \eta} (\rho u_o)$$

$$V1 = \frac{1}{\eta} \left[\frac{\partial}{\partial \eta} (\rho v_o w_o) - \frac{\partial}{\partial \eta} \left(\frac{\mu_e}{\eta} \frac{\partial v_o}{\partial \theta} \right) - \frac{\partial}{\partial \eta} \left(\mu_e \eta \frac{\partial}{\partial \eta} \left(\frac{w_o}{\eta} \right) \right) \right]$$

$$V2 = \left(1 - \frac{r_a}{\eta}\right) \left[\frac{1}{\eta} \frac{\partial}{\partial \theta} \left(\frac{\mu_e}{\eta} \frac{\partial v_o}{\partial \theta} + \mu_e \eta \frac{\partial}{\partial \eta} \left(\frac{w_o}{\eta} \right) \right) - \frac{2}{\eta} \left(\frac{\mu_e}{\eta} \frac{\partial w_o}{\partial \theta} + \frac{\mu_e}{\eta} v_o \right) - \frac{1}{\eta} \frac{\partial}{\partial \theta} (\rho v_o w_o) \right] \\ - \frac{r_a}{\eta^2} \mu_e \frac{\partial v_o}{\partial \eta} + \frac{1}{\eta} \frac{\partial}{\partial \eta} (2 \eta \mu_e \frac{\partial v_o}{\partial \eta}) - \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \rho v_o v_o) + \frac{r_a}{\eta^2} \rho v_o v_o + \frac{\eta - r_a}{\eta} \frac{\rho}{\eta} w_o w_o - \frac{\partial \rho_o}{\partial \eta}$$

$$V3 = (r_a - \eta) \frac{\partial}{\partial \eta} (\rho v_o)$$

$$W1 = \frac{1}{\eta} \left[\frac{\partial}{\partial \eta} (\rho w_o w_o) - \frac{\partial}{\partial \eta} \left(2 \frac{\mu_e}{\eta} v_o \right) - \frac{\partial}{\partial \eta} \left(2 \frac{\mu_e}{\eta} \frac{\partial w_o}{\partial \theta} \right) + \frac{\partial \rho_o}{\partial \eta} \right]$$

$$W2 = \left(1 - \frac{r_a}{\eta}\right) \left[\frac{2}{\eta} \frac{\partial}{\partial \theta} \left(\frac{\mu_e}{\eta} \frac{\partial w_o}{\partial \theta} + \frac{\mu_e}{\eta} v_o \right) + \frac{1}{\eta} \left(\frac{\mu_e}{\eta} \frac{\partial v_o}{\partial \theta} + \mu_e \eta \frac{\partial}{\partial \eta} \left(\frac{w_o}{\eta} \right) \right) - \frac{\rho}{\eta} v_o w_o - \frac{1}{\eta} \frac{\partial \rho_o}{\partial \theta} - \frac{1}{\eta} \frac{\partial}{\partial \theta} (\rho w_o w_o) \right] \\ + \frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\eta^2 \mu_e \frac{\partial}{\partial \eta} \left(\frac{w_o}{\eta} \right) \right) - \frac{r_a}{\eta} \mu_e \frac{\partial}{\partial \eta} \left(\frac{w_o}{\eta} \right) - \frac{r_a}{\eta} \frac{\mu_e}{\eta^2} \frac{\partial v_o}{\partial \theta} + \frac{1}{\eta} \frac{\partial}{\partial \eta} \left(\mu_e \frac{\partial v_o}{\partial \theta} \right) - \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta \rho v_o w_o) + \frac{r_a}{\eta^2} \rho v_o w_o$$

$$W3 = (r_a - \eta) \frac{\partial}{\partial \eta} (\rho w_o)$$

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